Charge carrier complexes in monolayer semiconductors

E. Mostaani,1 R. J. Hunt,2,3 D. M. Thomas,2 M. Szyniszewski,2,4 A. R.-P. Montblanch,5 M. Barbone,1,5 M. Atatüre,5 N. D. Drummond,2 and A. C. Ferrari1

1Cambridge Graphene Centre, University of Cambridge, 9 J. J. Thomson Avenue, Cambridge CB3 0FA, United Kingdom
2Department of Physics, Lancaster University, Lancaster LA1 4YB, United Kingdom
3Department of Engineering, Lancaster University, Lancaster LA1 4YB, United Kingdom
4Department of Physics and Astronomy, University College London, London WC1E 6BT, United Kingdom
5Cavendish Laboratory, University of Cambridge, 19 J. J. Thomson Avenue, Cambridge CB3 0HE, United Kingdom

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I. INTRODUCTION

The optical properties of layered semiconductors, such as transition-metal dichalcogenides (TMDs), change as the sample thickness is reduced from bulk (B) to a single layer (1L) [1]. Indirect band gaps in B-TMDs are often observed as the sample thickness is reduced from bulk (B) to a single layer [1]. This is due to the emergence of photoluminescence (PL) [3]. Excitonic effects are enhanced in 1L relative to B-TMDs, due to reduction in the dimensional (2d) nature of the electrostatic screening, as this modifies the form of the interaction between carriers [4]. In many 1L semiconductors, including TMDs with honeycomb lattices, spin-orbit coupling splits the conduction (CB) and valence (VB) bands at their extrema at the K and K’ points of the Brillouin zone [5–7]. This results in optically controllable spin and valley degrees of freedom [8–10]. Valley polarization is retained for >1 ns [8,11], ideal for quantum device applications, such as quantum light-emitting diodes [12–15]. Localized single-photon emitters that can be controlled by electroluminescence [12,15] are also promising for quantum photonics [15].

The binding energy (BE) of an exciton may be calculated from first principles by solving the Bethe-Salpeter equation (BSE) [16] on top of many-body perturbation theory calculations within the GW approximation [17–19], or by quantum Monte Carlo (QMC) methods [20]. However, studying charge-carrier complexes, such as quintons, using these approaches is computationally expensive [21]. Instead, the effective-mass approximation [22] can be used, whereby the ground-state energy is modelled by considering an electron (e) and a hole (h) interacting within a two-band model [23], and their effective masses are defined by experiment, or by first principles band structure calculations. In effective-mass models of charge-carrier complexes in 1L-TMDs with results obtained using ab initio interaction potentials. Magnetic fields <8 T change BEs by ~0.2 meV T−1, in agreement with experiments, with BE variations of different complexes being very similar. Our results will help identify charge complexes in the PL spectra of 1L semiconductors.

DOI: 10.1103/PhysRevB.108.035420

PHYSICAL REVIEW B 108, 035420 (2023)
and 2 h species available to form charge-carrier complexes at and below RT [29]. We consider all quasiparticles in quintons to be distinguishable, because in TMDs, e.g., they comprise 3 e and 2 h species distinguishable due to their different spin and/or valley degrees of freedom, as shown in Fig. 1(a). The reason why 2 of the 3 e are not considered as indistinguishable is that, with 2 available valleys in the CB [7], and 2 spins [7], and a spin-orbit splitting of the CB comparable to RT [33,37,57], so we only show complexes with distinguishable charge carriers, because they are stable and should be experimentally observable. Note that in (b) recombination may be followed by momentum exchange, giving an additional contribution of $2\Delta'$.

and 2 h species available to form charge-carrier complexes at and below RT [29]. We consider all quasiparticles in quintons to be distinguishable, because in TMDs, e.g., they comprise 3 e and 2 h species distinguishable due to their different spin and/or valley degrees of freedom, as shown in Fig. 1(a). The reason why 2 of the 3 e are not considered as indistinguishable is that, with 2 available valleys in the CB [7], and 2 spins [7], and a spin-orbit splitting of the CB comparable to RT [7], there are effectively 4 available e species in TMDs [7]. Quintons have 3 e distinguishable by valley and spin degree of freedom, as shown in Fig. 1(a), because they can occupy 3 of 4 available distinguishable levels. A biexciton with 2 indistinguishable e is unstable except at extreme mass ratios ($m_e/m_h > 3$), because exchange effects between the heavy particles are negligible, as shown in Ref. [29]. Adding 1 e to form a quinton will make the complex even less stable, because the size of the charge complex increases. Hence, we consider only the case of distinguishable e.

One can distinguish dark [39,40], bright [39,41], and semidark [42] charge-carrier complexes in TMDs. In dark complexes, Figs. 2(a) and 2(b), radiative e-h recombination is not allowed due to spin and/or momentum mismatch between the constituent e/h [43], while in bright complexes, Fig. 2(c), direct radiative e-h recombination is allowed by conservation of linear and angular momentum [44]. In semidark complexes, Fig. 2(d), radiative recombination can take place following an intervalley scattering event assisted by a phonon that maintains spin, but swaps an e, e.g., from valley $K'$ to $K$ [40–42,45], accompanied by an energy shift due to the change in occupation of the upper and lower spin-split bands [42].

Due to the nature of the CB spin-splitting of Mo-TMDs [7,46–48], Fig. 1(a), bright states are energetically lower than dark [46,49,50]. Hence, at low temperature $T < 100 K$, e in exciton (X), negative trion ($X^-$), and biexciton (XX) complexes occupy the lower spin-split bands. X complexes therefore travel only a small distance, e.g., $\sim 1 \mu m$ in 1L-MoTMDs [51,52], before radiative recombination, which reduces the chance to bind with another charge-carrier complex [53]. Furthermore, the XX PL peak may be difficult to distinguish from that of X$^-$, due to the small energy difference $\sim 10$ meV between their BEs [51]. Reference [37] detected XX and quintons (XX$^-$) in 1L-MoSe$_2$ by 2d coherent spectroscopy (2dCS) [54]. This method can focus on a delay time $\sim 10$ ps, over which XX or XX$^-$ are likely to form [55,56]. However, in 1L-W-TMDs, the most energetically stable excitonic states are dark [49], so that X have longer lifetimes ($\sim 1$ ps) [49] than in 1L-MoTMDs ($\sim 0.5$ ps) [49], favoring larger than X complexes. We thus focus on 1L-W-TMDs when comparing theory with experiments.

Figure 1 classifies XX$^-$ in 1L-Mo- and W-TMDs with respect to recombination energy and $T$-dependence of the emitted photons’ intensity. There are two XX$^-$ types: (1) those with 1e in the upper spin-split CB and 2e in the lower spin-split CB, and (2) those with 2e in the upper spin-split CB and 1e in the lower spin-split CB. The CB spin splittings in 1L-Mo- and W-TMDs are $\sim 3$ and $\sim 30$ meV [7], respectively. These are much less than the XX$^-$ BEs $\sim 50$ meV [33,37,57], as reported in Table I. The fact that the XX$^-$ BE is larger than the spin splitting implies XX$^-$ complexes are thermodynamically stable at $T$ close to 0 K, even taking into account the energy required to excite 1e to the upper spin-split CB. Assuming the CB spin-orbit splitting $\Delta'$ of 1L-TMDs to be $\ll XX^-$ BE, $E_{XX^+}$, each XX$^-$ can be treated as a two-state
system [58]. For $k_BT \ll \Delta', \ll E^b_{XX}$, with $k_B$ the Boltzmann’s constant, the fraction of $XX^-$ with $1e$ and $2e$ in the upper spin-split CB, hence the PL intensity of the corresponding $XX^-$, is:

$$I(T) \sim \begin{cases} \text{const.} & \text{for } 1e \text{ in upper spin band} \\ e^{-\Delta/(k_BT)} & \text{for } 2e \text{ in upper spin band} \end{cases} \quad (1)$$

Here, we use DMC within the effective-mass approximation to calculate $XX^-$ energies in 1L-LSMs. $XX^-$ are the largest free charge-carrier complexes in TMDs [29] at dilute limit (electron density $<1.5 \times 10^{12} \text{ cm}^{-2}$). In highly doped TMDs, larger charge-carrier complexes, such as six and eight body states, may form [59]. We provide an interpolation formula for $XX^-$ BEs for all 1L-LSMs as a function of $e$ and $h$ effective masses, permittivity of the surrounding media, and in-plane susceptibility of the 1L-LSM. We also use DMC to calculate the energies of charge-carrier complexes in the presence of out-of-plane magnetic and in-plane electric fields. Our studies of charge-carrier complexes in external fields help understand and identify them, e.g. in experimental PL spectra of TMDs. This helps to identify whether the behavior in external fields can be used to investigate PL peaks. We find that applying an external magnetic field helps identifying charge-carrier complexes in 1L-LSMs with different $e$ and $h$ effective masses, while electric fields can be used to identify charge-carrier complexes in all 1L-LSMs.

We explore the accuracy of the RKI potential by comparing BEs with results obtained using $ab initio$ random-phase approximation (RPA) interaction potentials [60]. We find that, within the effective-mass approximation, RKI can describe quasiparticles on length scales larger than the lattice constant. Thus, our results can be used to determine the PL of excitonic complexes.

### II. RESULTS AND DISCUSSION

#### A. Units

In the followign, we will use Hartree excitonic units (e.u.), in which the e-h reduced mass $\mu$, $4\pi$ times the absolute permittivity $\epsilon$, the Dirac constant $\hbar$, and the charge $e$ are all equal to $1$, i.e., $\mu = 4\pi \epsilon = \hbar = e = 1$. This helps to scale the BEs with respect to $r_*$ and effective masses, as explained in Methods. The screening length is $r_s = 4\pi \epsilon = \hbar = e = 1$. That of energy is the exciton Bohr radius $a_0^* = 4\pi e \hbar^2/(\mu \epsilon^2)$ [67], of magnetic flux density is $B^* = \mu^2 e^3/(4\pi \epsilon) \hbar^2$, of electric field is $F^* = \mu^2 e^3/(4\pi \epsilon) \hbar^2$, and that of energy is the exciton Rydberg constant $R_\infty^* = \mu e^4/(2\pi \epsilon^2 \hbar^2)$, the exciton Rydberg constant [67].
For the logarithmic approximation to the RKI we will use a different set of units, as explained in Ref. [29]. Since in the logarithmic regime \( r \ll r_0 \), where \( r \) is the separation between charge carriers, the behavior of the energy changes when compared with the intermediate regime \( r \gg r_0 \). In the logarithmic e.u., the e-h reduced mass \( \mu, 4\pi \varepsilon r_s, \hbar \), and the electronic charge are all equal to 1, i.e., \( \mu = 4\pi \varepsilon r_s = \hbar = e = 1 \). We define the logarithmic e.u. of length to be \( \sqrt{2r_0} \), where \( r_0 = \sqrt[4]{4\pi \varepsilon r_s \hbar^2/(2\varepsilon^2 \mu)} \), the unit of energy \( E_0 = e^2/(4\pi \varepsilon r_s) \), the unit of magnetic flux density \( B_0 = \sqrt[4]{\mu E_0}/(\sqrt{2}\varepsilon r_0) = e\mu/(4\pi \varepsilon r_s \hbar) \), and the unit of electric field \( F_0 = E_0/(\sqrt{2}\varepsilon_0 e) = e^2\mu/[(4\pi \varepsilon r_s \hbar)^2] \).

To convert the Bohr radius, flux density, electric field, and energy from e.u. to SI units, each value needs to be multiplied by \( a_0^n, B^*, F^*, \) and \( 2R_s^* \), respectively. To convert from logarithmic e.u. to SI units, each value needs to be multiplied by \( \sqrt{2r_0}, B_0, F_0 \), and \( E_0 \), respectively.

Table II summarizes all acronyms in this paper [68].

### B. Binding energies

We define the \( X, X^- \), and \( XX \) BES as:

\[
E_X^b = E_e - E_h - E_X, \quad (2)
\]
\[
E_X^b = E_e + E_X - E_X^-, \quad (3)
\]
\[
E_{XX}^b = 2E_X - E_{XX}, \quad (4)
\]

where the complexes are defined in Table II. In the absence of external fields, \( E_e = E_h = 0 \).

We define the de-excitonization energy of \( XX^- \) as:

\[
E_{XX^-}^{DE} = E_X + E_{XX^-} - E_{XX^-}, \quad (5)
\]

and the electron affinity of \( XX \) as:

\[
E_{XX}^{EA} = E_{XX} + E_e - E_{XX^-} = E_X^b - E_{XX}^b + E_{XX}^{DE}. \quad (6)
\]

Since the most stable dissociated complexes have the lowest ground-state energies, the \( XX^- \) BE is the minimum of \( E_{XX}^{DE} \) and \( E_{XX}^{EA} \) for a given \( r_s \) and effective mass:

\[
E_{XX^-}^b = \min \{ E_{XX}^{DE}, E_{XX}^{EA} \}. \quad (7)
\]

Comparing Eq. (7) with Eqs. (5) and (6) for 1L-TMDs, the energy difference between bright \( X \) and \( XX^- \) PL peaks is \( E_{XX^-}^{DE} \).

We can calculate \( E_{XX}^{DE} \) of \( XX^- \) complexes in all 1L-LSMs with all possible values for \( r_s/a_0^n = \{0, 0.5, 1, 2, 4, 6, 8, \infty\} \) and \( \sigma = \{0, 0.1, 0.2, \ldots, 1, 1.5, 4, 9, \infty\} \), where \( \sigma = m_e/m_h \) is the mass ratio. We fit:

\[
E_{XX}^{DE} = \frac{\sum_{i=1}^4 \sum_{j=1}^{5-i} a_i j^r \beta_j + b_1 \sqrt{x}}{1 + \sum_{k=1}^3 c_k \sqrt{x}} \quad (8)
\]

where \( a_{ij} \), \( b_{ij} \), \( c_k \), and \( d_k \) are fitting parameters, \( \chi = \sigma/(\sigma + 1) = m_e/(m_e + m_h) \) is a rescaled mass ratio, and \( \gamma = r_s/(r_s + a_0^n) \) is a rescaled in-plane susceptibility parameter. The fitting function goes as the square root of the mass at extreme mass ratios (\( \sigma = 0 \) and \( \sigma = \infty \)), as required by the Born-Oppenheimer approximation [67]. We fit \( E_{XX^-}/[R_s^*(1 - y)] \) so that the asymptotic behavior at \( r_s \to \infty \)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>2d</td>
<td>two-dimensional</td>
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<tr>
<td>TMD</td>
<td>Transition metal dichalcogenide</td>
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<tr>
<td>1L</td>
<td>Monolayer</td>
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<tr>
<td>ML</td>
<td>Multilayer</td>
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<tr>
<td>LSMS</td>
<td>Layered semiconductor materials</td>
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<td>VB</td>
<td>Valence band</td>
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<td>CB</td>
<td>Conduction band</td>
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<td>PL</td>
<td>Photoluminescence</td>
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<td>μ</td>
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<td>m_e</td>
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<td>Hole effective mass</td>
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<td>CoM</td>
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<td>ε</td>
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<td>h</td>
<td>Dirac constant</td>
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<td>r_s</td>
<td>Screening length</td>
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<td>B</td>
<td>Magnetic flux density</td>
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<td>F</td>
<td>Electric field</td>
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<td>R_y</td>
<td>Exciton Rydberg constant</td>
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<td>a_0^n</td>
<td>exciton Bohr radius</td>
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obtained using the logarithmic interaction can be included in the fit. The error in the fitted \( E_{\text{DE}}^{\text{XX}} \) is < 5% at each data point. The statistical error bars on the DMC \( E_{\text{DE}}^{\text{XX}} \) data points are much smaller than the error in the fit. We therefore use an unweighted least-squares fit [69]. We provide a program [70] that can be used to evaluate \( E_{\text{DE}}^{\text{XX}} \) and \( \Delta E \) for any 1L-TMDs, for which effective masses, \( r_\ast \), and dielectric constant of the environment are the inputs.

Using the BE fits in Eqs. (48) and (49) of Ref. [29] together with Eqs. (6)–(8), we calculate the XX BEs in Fig. 3. Above the yellow line in Fig. 3(c), the XX BE “\( \Delta E_{\text{DE}}^{\text{XX}} \)” (all 1L-TMDs fall in this region). Below the yellow line, the XX BE is equal to the XX electron affinity.

Table I lists the XX BEs from DMC and the fit of Eq. (8) for 1L-WS2, 1L-WSe2, 1L-WTe2, 1L-MoS2, 1L-MoSe2, and 1L-MoTe2.

To measure \( \Delta E \), XX, \( \Delta E \) BEs, in Ref. [33] we used continuous wave (CW) PL at 4K for 1L-WSe2 encapsulated between two 10 nm (bottom) and 3 nm (top) ML-hBN on Si/SiO2. Refs. [33,38,57] employed different experimental conditions for 1L-WSe2, but they all produced similar results, Table I, across various techniques and substrates.

The size of a charge-carrier complex in a 1L-TMD can be defined by \( r_0 \). This is < 8 Å for the TMDs listed in Table I, because their \( e \) and \( h \) masses and screening lengths are around the same order of magnitude. Hence, we suggest that encapsulation in > 1 nm ML-hBN can be described by the permittivity \( \epsilon = 4\epsilon_0 \) [71–75] of bulk hBN.

For ML-hBN-encapsulated TMDs, we test 2 approaches to compare our results with experiments.

(1) We fix \( \epsilon = \epsilon_0 \) and determine \( r_\ast \) by fitting Eqs. (48) and (49) of Ref. [29] to the experimental \( \Delta E \), XX BEs in Refs. [33,37,38,57]. This is reasonable because, at distances larger than the layer-layer separation, the Keldysh interaction of Eq. (15) for a ML is of the same form as for a 1L [76], but with \( r_\ast \) being the sum of \( r_\ast \) for the different layers [76]. For \( r \ll r_\ast \), only \( \epsilon \) appears in the logarithmic approximation to the Keldysh interaction in Eq. (16), apart from a constant contribution to the total energy, which cancels out of \( E_{\text{DE}}^{\text{XX}} \). Hence, it is preferable to fix \( \epsilon \) and treat \( r_\ast \) as the independent parameter. The XX BEs calculated with this approach for 1L-WSe2 and 1L-WSe2 encapsulated in ML-hBN agree with the experiments in Refs. [33,57], differing by at most ~ 2 meV, as for Table I.

(2) We use \( ab \) initio vacuum \( r_\ast \) for 1L-TMDs. To describe hBN encapsulation, we use \( \epsilon = 4\epsilon_0 \), consistent with Refs. [71–74]. This gives BEs ~ 5–18 meV smaller than Refs. [33,57]. This difference could either be due to the phenomenological parameters (\( r_\ast \) and \( \epsilon \)), obtained by \( ab \) initio methods and used in the Mott-Wannier-Keldysh model of Eq. (14), or be a result of neglecting intervalley scattering [77] and contact (exchange) interactions.

The substrate effect on the BE of a charge-carrier complex can be described by:

\[
\epsilon = (\epsilon_0 + \epsilon_{\text{substrate}})/2, \quad (9)
\]

where \( \epsilon_{\text{substrate}} \) is the bulk permittivity of the substrate. In Ref. [38], 1L-WSe2 was grown on SiO2 by physical vapor deposition and the \( \Delta E \) and XX BEs of 1L-WSe2 at RT were measured as ~ 30 and 51 meV, respectively, by CW PL. Using the permittivity of SiO2, \( \epsilon_{\text{substrate}} \sim 3.9\epsilon_0 \) [78] in Eqs. (9) and (8), and Eq. (49) of Ref. [29], we calculate the \( \Delta E \) and XX BEs to be ~ 19 and 38 meV, respectively, ~ 10–13 meV less than the experiments in Ref. [38] done at RT, while our calculations correspond to \( T = 0 \) K. Approximating the sapphire permittivity as isotropic with \( \epsilon_{\text{substrate}} = 8.9\epsilon_0 \) [79] gives \( \Delta E \), XX, XX BEs

FIG. 3. (a) DMC BEs of XX as a function of \( r_\ast/(r_\ast + a_\ast^L) \). (b) DMC BEs of XX as a function of \( \sigma/(\sigma + 1) \). (c) XX BEs as a function of rescaled susceptibility and mass ratio. Above the yellow line \( E_{\text{DE}}^{\text{XX}} < \Delta E \) electron affinity, so that X and XX are the most energetically competitive. Below the yellow line the situation is reversed, so that XX and free e are the most competitive. The white stars show the mass ratios and in-plane susceptibility of 1L-MoSe2 at (0.45, 0.93), 1L-MoSe2 at (0.46, 0.94), 1L-MoTe2 at (0.50, 0.98), 1L-WSe2 at (0.46, 0.91), 1L-WSe2 at (0.46, 0.93), 1L-WTe2 at (0.41, 0.95), where the first and second numbers in brackets are \( \sigma/(\sigma + 1) \) and \( r_\ast/(r_\ast + a_\ast^L) \), respectively. XX BEs are between 0.00736(5)\( R^0 \) and 0.0288(1)\( R^0 \), with the numbers in brackets the BE error bars.
in 1L-MoSe$_2$ to be $\sim 13.6$, $13.7$, and $30$ meV, respectively. We use an effective dielectric constant to describe the environment for 1L-TMDs on a substrate. This is evaluated as the average of the dielectric constants of the media above and below the 1L-TMD. For 1L-TMD on SiO$_2$, we have SiO$_2$/TMD/vacuum, hence $\epsilon = \epsilon_0(3.9 + 1)/2 = 2.45\epsilon_0$. For sapphire/1L-TMD/vacuum, $\epsilon = \epsilon_0(8.9 + 1)/2 = 4.95\epsilon_0$. In Ref. $[37]$, exfoliated 1L-MoSe$_2$ was transferred to a sapphire substrate and 2dCS at 13 K was used to measure $X^-$, XX, and $XX^-$ BEs $\sim 27$, 18, and 40meV, respectively $[37]$. Substrate-induced roughness can also cause inhomogeneity in the electronic structure and extra carrier scattering $[80]$. This affects PL, leading to inhomogeneous broadening $[81,82]$, which makes it difficult to identify charge-carrier complexes $[37]$. In Ref. $[37]$, PL spectra were not recorded as a function of excitation power. However, power-dependent measurements help assign the PL peaks to $XX$, $XX^-$, and $XX^{\ast}$ for encapsulated 1L-TMDs does not depend only on the intrinsic properties of the TMD under investigation, but also on the surrounding dielectric media $[76]$ (and their relative permittivities $[76]$), and, ultimately, on the electronic structure of the interfacial regions $[83]$ (whose dielectric response arises from their own electronic structure $[76]$). To simplify calculations, we fix $\epsilon$ to the vacuum permittivity and vary $r_s$ as shown in Table I. Due to the complexity in defining $r_s$ and $\epsilon$ for 1L-TMDs encapsulated in hBN or placed on a substrate, we use the first approach, where we fit $\epsilon = \epsilon_0$ and determine $r_s$ by fitting theoretical $X^-$ and XX BEs to available experiments, so to define the $XX^-$ BEs. The reason is that calculating the dielectric constant and $r_s$ of encapsulated materials using first-principles requires full understanding of the interfacial and interlayer interactions. Here, we primarily focus on freestanding 1L-LSMs, and then extend our model for encapsulated 1L-LSMs. The components that are missing in the calculations (such as e-h exchange) may be of higher relevance, but the Mott-Wannier model with the Keldysh interaction provides a good description of the energies of excitons, trions, and biexcitons in 1L-LSMs, as discussed in Refs. $[28,29,84]$, and there is no compelling reason to believe that contact interactions should be more important in a quantum than in a trion or biexciton in freestanding 1L-LSMs. XX with two indistinguishable e are unstable in 1L-TMDs, because of the resulting antisymmetry of the spatial wave function. We concluded that, in bound complexes featuring only singly charged dopant ions and charge carriers, all charge carriers must be distinguishable. Our results show that a charge-carrier complex can feature at most one dopant ion. Because of the band structure (see Fig. $1(a)$), 1L-TMDs can have 4e species and 2h species. This suggests that the largest stable cluster will have a positive donor ion, 4 distinguishable e, and 2 distinguishable h. We get bound-state wave functions describing the donor-bound double-negative XX ($D^{\ast}XX$). These seven-body complexes are predicted to be stable in 1L-WS$_2$, 1L-WS$_2$, 1L-MoS$_2$, 1L-MoS$_2$ in vacuum and air. The DMC-calculated BEs with respect to the most energetically favorable products [donor-bound negative X ($D^{\ast}X^{\ast}$)+free X] are in Table III. Because the dominant decay products include an X, the BE gives the PL peak position of the $D^{\ast}XX$ complex relative to the X line, possible in samples containing donor defects.

### D. Accuracy of the Rytova-Keldysh interaction

RKI arises from the approximation that the in-plane susceptibility of a material is a constant $[24]$. The potential we use is always $ab$ initio RPA. $Ab$ initio calculations were performed to determine noninteracting band structures of TMDs (via DFT) in Ref. $[60]$, used here, in conjunction with the RPA assumption, to form a dielectric function. We do not have screening of Coulomb interactions by free e (metallic behavior). Instead, we have screening of Coulomb interactions by bound e (insulating/semiconducting behavior). Indeed, at long range ($r \gg r_s$), the potentials we examine reduce to the unscreened Coulomb interaction (or, in the presence of a dielectric environment, to an isotropic, statically screened Coulomb interaction). Here, we investigate the RKI accuracy by using an alternative approach based on $ab$ initio calculations for 1L-MoS$_2$. Realistic dielectric functions exhibit spatial dependencies which differ from the Coulomb interaction at short range ($r \ll r_s$). At long range ($r \gg r_s$), in-plane screening becomes irrelevant, and all physical dielectric functions behave as the Coulomb interaction, as explained in Methods. Given that the binding of excitonic complexes occurs on length scales larger than the lattice spacings (Table I), where screening effects are most prominent (Fig. 4), an investigation into their effects on charge-carrier binding is warranted. Reference $[60]$ parameterized a dielectric permittivity $\epsilon(q)$ for 1L-MoS$_2$ via RPA applied to Kohn-Sham orbitals from density functional theory calculations to study charged defects. We refer to the real-space interaction formed

### Table III. Theoretical $E_D^{XX}$ for 1L-TMDs in vacuum, with $ab$ initio masses and $r_s$ from Table I

<table>
<thead>
<tr>
<th>TMD</th>
<th>$E_D^{XX}$ (meV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1L-MoS$_2$ (vac.)</td>
<td>58.3(5)</td>
</tr>
<tr>
<td>1L-MoSe$_2$ (vac.)</td>
<td>78.6(3)</td>
</tr>
<tr>
<td>1L-WS$_2$ (vac.)</td>
<td>80.4(5)</td>
</tr>
<tr>
<td>1L-WSe$_2$ (vac.)</td>
<td>70.5(7)</td>
</tr>
</tbody>
</table>
from $\epsilon(q)$ as the RPA interaction (RPAI), and compare it to RKI in Fig. 4, which plots the potentials in real space.

We use cusp conditions [21] to prevent the wave function of charge carriers to diverge around particle coalescence points [29]. We use the same trial-wave-function form as our calculations with RKI, see Methods for details. As a test, Fig. 5 verifies we reproduce the theoretical donor-atom BEs of Ref. [60], for an adatom-bound $e$ above a 1L-MoS$_2$ surface. Our data have small $\sim 10^{-4}$--$10^{-3}$ meV error bars, and differ from Ref. [60] by a few meV, for typical BEs $\sim$ a few hundreds meV.

Figure 4 indicates that at distances $> a_{1L-MoS_2}$, RPAI follows the same form as RKI (and, ultimately, Coulomb) interactions. However, at distances $\sim a_{1L-MoS_2}$, RKI no longer overlaps RPAI, hence cannot describe the interaction between quasiparticles. Figure 4 shows that for $r \ll 0.5 a_{1L-MoS_2}$, RPAI reduces to an unscreened $1/r$ potential, while the RKI behavior is that of a logarithmic divergence [25]. However, within the effective-mass approximation, we can only describe quasiparticles on length scales $> a_{1L-MoS_2}$, as shown in Table I and Fig. 4, whose associated Bloch wave packets are localized in momentum space, with well-defined effective mass.

The RPAI BEs of charge-carrier complexes are in Table IV. Removing the bare Coulomb interaction at distances $< a_{1L-MoS_2}$ is necessary to obtain results in agreement with previous experimental [86,87] and theoretical [88] works. For $r_c < a_{1L-MoS_2}$, we truncate the RPAI to a constant $v(r < r_c) = v(r_c)$. The precise value of $r_c$ is not particularly important for the BE calculation of charge-carrier complexes, as we observe a weak BE dependence on it, see Table IV.

Table IV indicates that there is no need to use an expression for the electrostatic interaction between charge carriers in LSMs more sophisticated than RKI when evaluating BEs of trions, biexcitons, and quintons. As explained in Methods, any errors in the Mott-Wannier-Keldysh model of charge-carrier complexes for isolated 1L are either due to the parameters (effective masses, $r_s$, environment permittivity), or to a more fundamental breakdown of the effective-mass approximation. Interv valley scattering may play an important role in the complexes’ BEs [60], while exchange effects could be relevant in highly localized complexes [89].

**E. Complexes in uniform magnetic fields**

For an out-of-plane external magnetic field of flux density $B = (0, 0, B)$, where $B$ is a positive constant, we can write the Hamiltonian as:

$$
\hat{H} = \sum_i \frac{1}{2m_i} (-i\hbar \nabla_i - q_i \mathbf{A}_i)^2 + \sum_{i>j} \frac{q_i q_j}{4\pi \epsilon r_{ij}} V(r_{ij}/r_s) $$

$$
= \sum_i \left( \frac{-\hbar^2}{2m_i} \nabla_i^2 + \frac{\hbar q_i}{m_i} \mathbf{A}_i \cdot \nabla_i + \frac{q_i^2 |\mathbf{A}_i|^2}{2m_i} \right) + \sum_{i>j} \frac{q_i q_j}{4\pi \epsilon r_s} V(r_{ij}/r_s),
$$

(10)

where $\mathbf{A}_i = -r_i \times \mathbf{B}/2 = -(y_i, x_i, 0)B/2$ is the magnetic vector potential for particle $i$ in the Coulomb gauge (so that $\nabla_i \cdot \mathbf{A}_i = 0$) [67]. We neglect the charge carriers’ intrinsic magnetic dipole moment energy in the external magnetic field, because this contribution cancels out.
FIG. 6. Theoretical BEs of (a) X, (c) X$^-$, and (e) XX as a function of perpendicular magnetic field for 1L-WSe$_2$ in vacuum. We use the ab initio mass and $r^*$ parameters of Table I. The CoM contribution for X is $E_{\text{CoM}}^X = (E^X_B)_B = 0 + E^e + E^h - E^\text{CoM}_X$; for X$^-$ is $E_{\text{CoM}}^{X^-} = (E^{X^-}_B)_B = 0 + E^e - E^\text{CoM}_X$; and for XX is $E_{\text{CoM}}^{XX} = (E^{XX}_B)_B = 0 + 2E^e - E^h - E^\text{CoM}_XX$. Experimental BEs of (b) X, (d) X$^-$, (f) XX for 1L-WSe$_2$ encapsulated in hBN, compared with DMC ones using $\epsilon = \epsilon_0$ and $r^* = 48$ Å and the fit to Eq. (21).

Substituting $\mathbf{A}_i$ into Eq. (10), the term $q_i^2 |\mathbf{A}_i|^2 / (2m_i) = q_i^2 B^2 |\mathbf{r}_i|^2 / (8m_i)$ provides a quadratic confining potential for the particles in the complex. This cannot be regarded as a perturbation for the (otherwise free) center-of-mass (CoM) motion, because there is a quantitative difference between a bound state wave function in a quadratic potential and free motion in zero potential, no matter how small the quadratic coefficient [31]. The zero-point energy of the CoM motion in the confining potential results in a linear $[O(B)]$ contribution to the total energy, as given in Eq. (12). The term also weakly perturbs the relative motion within the complex, giving a quadratic $[O(B^2)]$ contribution to the energy. We thus include the $q_i^2 |\mathbf{A}_i|^2 / (2m_i) = q_i^2 B^2 |\mathbf{r}_i|^2 / (8m_i)$ term in our QMC calculations. The linear $(i\hbar q_i/m_i)\mathbf{A}_i \cdot \nabla_i$ term in Eq. (10) breaks time-reversal symmetry as it is imaginary [90]. It only adds to the energy in second-order perturbation theory, giving another $O(B^2)$ contribution. This vanishes when we use a variational Ansatz consisting of a real trial wave function. We therefore neglect it.

The ground-state energies of isolated e/h are $E_e = \hbar eB/(2m_e)$ and $E_h = \hbar eB/(2m_h)$, in the presence of a magnetic field [67]. More generally, if a bound complex of $N_e$ e and $N_h$ h moves in a magnetic field, from Eq. (10) the quadratic confining potential is:

$$U = \sum_i \frac{e^2 B^2 |\mathbf{r}_i|^2}{8m_i} \approx \frac{B^2 e^2}{8} \left( \frac{N_e}{m_e} + \frac{N_h}{m_h} \right) R^2, \quad (11)$$
where $\mathbf{R}$ is the CoM position. The total mass of the complex is $N_e m_e + N_h m_h$. Hence, we obtain the CoM zero-point energy of a charge complex as:

$$E^\text{CoM} = \frac{\hbar e B}{2} \sqrt{\frac{N_e/m_e + N_h/m_h}{N_e m_e + N_h m_h}}.$$ (12)

If $m_e = m_h \equiv m$ then $E^\text{CoM} = \frac{\hbar e B}{2m}$, independent of $N_e$, $N_h$. For a bound complex, our results show that the magnetic field can always be made sufficiently weak so that the external potential is slowly varying on the length scale of the complex (i.e., $\sqrt{\hbar e B F_0} < a_0^2$). Hence, Eq. (12) is the leading-order contribution to the free charge-carrier complex energy in a magnetic field.

Figure 6 plots the DMC X, X$^-$, XX BEs for 1L-WSe$_2$ in vacuum, in the presence of an out-of-plane magnetic field, using RKI. $m_e$, $m_h$, and $r_\alpha$ are taken from Table I. Our results are in agreement with Ref. [91]. The CoM contribution of Eq. 12 is a good approximation to calculate the X, X$^-$, XX BEs in magnetic fields < 8 T, because it is exact up to linear order in magnetic field, within the effective-mass approximation. For the X BE in magnetic fields > 8 T, we use Eq. (21), derived in Methods. The fitted $C$ in Eq. (21) is 0.557 for X in 1L-WSe$_2$.

Figures 6(b), 6(d) and 6(f) compare our DMC BEs with measurements for hBN-encapsulated 1L-WSe$_2$. The sample is produced by exfoliating flux zone grown B-WSe$_2$ [92], then encapsulating it with ML-hBN (10 nm bottom and 3 nm top) using an all-dry technique [93,94]. Measurements are done in a closed-cycle cryostat (Attocube Attodry 1000) at 4 K, using an all-dry technique [93,94]. Measurements are done in a closed-cycle cryostat (Attocube Attodry 1000) at 4 K, with superconducting magnets allowing out-of-plane magnetic fields up to 8 T. CW excitation is provided with a diode laser at 658 nm, close to the 1L-WSe$_2$ optical band gap [95]. Polarization-resolved excitation and collection pass through a confocal microscope with the sample in reflection geometry. The PL signal is sent to a liquid-N$_2$-cooled spectrometer (Princeton).

We assume $r_\alpha = 48$ Å and $\epsilon = \epsilon_0$, as discussed in Sec. II B.

The theoretical and experimental BEs differ <0.3 meV over the 0–8 K range. The $O(B)$ magnetic-field dependence is only via the effective masses and $N_e$, $N_h$, via the CoM energy, Eq. (12). The fact that the theoretical and experimental magnetic-field trends in Fig. 6 agree well demonstrates that the approximation with $ab\ initialo$ effective masses is accurate. The main challenge is to obtain a sufficiently accurate interaction between charge carriers. The BE $O(B)$ term is the same for all complexes, in the limit $m_e = m_h$. For most 1L-TMDs, $m_e$ and $m_h$ are similar, Table I, implying that the magnetic-field dependence cannot be used to distinguish carrier complexes. Table V has DMC and experimental X, X$^-$, XX BEs for 1L-WSe$_2$ in the presence of an out-of-plane magnetic field, as for Fig. 6. The variation of BEs of different charge complexes is the same <8 T.

### F. Complexes in uniform electric fields

A bias voltage $\Delta V$ applied to a 1L-LSM results in an in-plane electric field. Its precise form depends on device geometry. Here, we assume a uniform electric field $F = -\Delta V/d$, where $d$ is the distance between terminals, for simplicity. $F$ will perturb the energies of charge-carrier complexes in the CoM frame. We therefore investigate the effects of $F$ on BEs by including an additional term $-\sum_i q_i F x_i$ in the Hamiltonian, where $x_i$ is the $x$ coordinate of particle $i$. Figure 7 plots the X BE shift as a function of electric field strengths for 1L-MoS$_2$, 1L-MoSe$_2$, 1L-WSe$_2$, 1L-WSe$_2$, in vacuum and encapsulated by hBN, using the $ab\ initialo$ parameters in Table I and $\epsilon = 4\epsilon_0$. In each case, the X BE goes as the square of the in-plane electric field, as expected for a linearly polarizable exciton [96]. Thus the total energy of an isolated neutral complex of polarizability $\alpha$ in a uniform $F$ is:

$$E = E_{F=0} - \alpha F^2/2,$$ (13)

where $E_{F=0}$ is the energy of the complex in the absence of external fields. The variation of energy with electric field strength remains quadratic up to at least ~50 mV nm$^{-1}$. At electric fields >50 mV nm$^{-1}$ we find that optimizing wave functions by VMC energy minimization does not result in bound-state wave functions. If the parameters in the wave function are fixed such that a bound state is forced, the resulting DMC calculations are unstable. It is possible that some, or all, complexes remain bound at these larger electric fields, and our QMC calculations become unstable, due to the choice of trial wave function. The form we use is isotropic, so it does not allow the complex to polarize in VMC. Polarization arises at DMC level.

<table>
<thead>
<tr>
<th>TMD</th>
<th>Polarizability (eV nm$^{-2}$ V$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
</tr>
<tr>
<td>1L-MoS$_2$ (vac.)</td>
<td>5.84(2)</td>
</tr>
<tr>
<td>1L-MoSe$_2$ (vac.)</td>
<td>5.76(2)</td>
</tr>
<tr>
<td>1L-WSe$_2$ (vac.)</td>
<td>8.04(3)</td>
</tr>
<tr>
<td>1L-MoS$_2$ (hBN)</td>
<td>17.17(4)</td>
</tr>
<tr>
<td>1L-MoSe$_2$ (hBN)</td>
<td>10.10(4)</td>
</tr>
<tr>
<td>1L-WSe$_2$ (hBN)</td>
<td>27.16(4)</td>
</tr>
<tr>
<td>1L-WSe$_2$ (hBN)</td>
<td>30.43(4)</td>
</tr>
</tbody>
</table>
FIG. 7. DMC BE shift for (a) X, (b) XX, (c) donor atoms as a function of $F^2$ for different 1L-TMDs in vacuum and encapsulated in hBN. Error bars in (a) and (c) are smaller than the symbols. The solid and dashed lines are BEs determined by the polarizabilities in Table VI for 1L-TMDs in vacuum and encapsulated by hBN. The vertical dotted lines correspond to $F = 50$ mV nm$^{-1}$, beyond which VMC energy minimization does not result in bound-state wave functions.

The XX and donor-atom BEs vary linearly with $F^2$, Fig. 7. However, while the donor-atom BEs increase with $F^2$, the XX BEs decrease. For a 4-particle complex, alignment of charges in the direction of the applied field places like charges closer together, and reduces BE with respect to dissociation into 2-particle complexes. Trion BEs also vary linearly with $F^2$, Fig. 8. However, QMC calculations become unstable at much lower $F$. This is reflected in higher polarizabilities for trions than for neutral complexes, Table VI.

The predicted BE shifts of each of the complexes are in Table VII for 1L-TMDs, both in vacuum and encapsulated by hBN, subject to $F = 50$ mV nm$^{-1}$, beyond which VMC energy minimization does not result in bound-state wave functions. The shifts in the peaks of the trions are so large that, at the very least, they should be experimentally distinguishable from the neutral complexes when an electric field is applied. Identification of a positive from a negative trion may be possible in some materials/environments, but not all. For neutral complexes, the differences of a few meV in BE shifts suggest they are unlikely to be experimentally identified by their peak shifts under an electric field.
TABLE VII. Calculated BE shifts of X, XX, D⁰, X⁻, X⁺ using Eq. (13) and polarizabilities in Table VI for 1L-TMDs, both in vacuum and encapsulated by hBN, for F = 50 mV nm⁻¹. Not all complexes are bound at F = 50 mV nm⁻¹.

<table>
<thead>
<tr>
<th>TMD</th>
<th>X</th>
<th>XX</th>
<th>D⁰</th>
<th>X⁻</th>
<th>X⁺</th>
</tr>
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<tbody>
<tr>
<td>1L-MoS₂ (vac.)</td>
<td>7.3</td>
<td>&lt;1</td>
<td>3.5</td>
<td>76</td>
<td>48</td>
</tr>
<tr>
<td>1L-MoSe₂ (vac.)</td>
<td>7.2</td>
<td>&lt;1</td>
<td>3.4</td>
<td>93</td>
<td>49</td>
</tr>
<tr>
<td>1L-WS₂ (vac.)</td>
<td>10.1</td>
<td>&lt;1</td>
<td>4.6</td>
<td>125</td>
<td>80</td>
</tr>
<tr>
<td>1L-WS₂ (hBN)</td>
<td>12.6</td>
<td>5.8</td>
<td>4.9</td>
<td>150</td>
<td>135</td>
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<tr>
<td>1L-MoSe₂ (hBN)</td>
<td>21.5</td>
<td>&lt;1</td>
<td>8.1</td>
<td>201</td>
<td>180</td>
</tr>
<tr>
<td>1L-MoS₂ (hBN)</td>
<td>20.3</td>
<td>&lt;1</td>
<td>8.6</td>
<td>243</td>
<td>206</td>
</tr>
<tr>
<td>1L-WS₂ (hBN)</td>
<td>34.0</td>
<td>&lt;1</td>
<td>6.2</td>
<td>362</td>
<td>273</td>
</tr>
<tr>
<td>1L-WSe₂ (hBN)</td>
<td>38.0</td>
<td>&lt;1</td>
<td>6.6</td>
<td>473</td>
<td>408</td>
</tr>
</tbody>
</table>

III. CONCLUSIONS

We used DMC to calculate XX⁻ BEs in 1L-LSMs within the effective-mass approximation, using the RKI potential. A program available online [70] can be used to evaluate interpolated XX⁻ BEs given e and h effective masses, in-plane susceptibility, and environment permittivity for a desired 1L-LSM. The BEs of charge-carrier complexes in 1L-LSMs in vacuum from RKI are in excellent agreement with those obtained using interaction potentials taken from ab initio RPA, suggesting RKI is a reliable interaction potential to describe screened interaction between charge carriers in 1L-LSMs.

We also considered the effect of external out-of-plane magnetic fields and in-plane electric fields on BEs of charge-carrier complexes in 1L-LSMs. The resulting BE changes are linear in magnetic fields and quadratic in electric fields up to 10 T and 50 mV nm⁻¹.

We measured X, X⁻, XX BEs for hBN-encapsulated 1L-WSe₂ up to 8 T, where the BEs vary linearly with magnetic field, and found them to be in good agreement with the effective-mass approximation using ab initio effective masses. These BE shifts could in principle be used to identify complexes in PL experiments, provided m⁺ and m⁻ are different. In practice, m⁺ and m⁻ in 1L-TMDs are too similar to distinguish complexes in external magnetic fields. In-plane electric fields should shift the BE peaks in proportion to the field strength and allow for identification of charged from neutral complexes.

We derived BEs of charge-carrier complexes in 1L-TMDs by solving the interacting quantum few-body problem for each complex, working within the effective-mass approximation, with a RKI potential between charge carriers. The BE magnetic-field dependence agrees with experiments on a sub-meV energy scale. Since this only involves m⁺ and m⁻, and not the parameters describing the screened interaction, the approximation with ab initio effective masses is highly accurate.

IV. METHODS

A. Effective-mass approximation

All our calculations are performed within the effective-mass approximation. For charge-carrier complexes in 1L-LSMs in the absence of external fields, we solve the Mott-Wannier-Keldysh Schrödinger equation [29]:

$$\psi = E \psi,$$  \hspace{1cm} (14)

where $V(r/r_*) = \frac{\pi}{2} [H_0(r/r_*) - Y_0(r/r_*)],$  \hspace{1cm} (15)

where $H_0(\kappa)$ is a Struve function [98] and $Y_0(\kappa)$ is a Bessel function of the second kind [98]. At long range ($r \gg r_*$) the potential in Eq. (15) is a Coulomb interaction $V(r/r_*) \sim r_*/r$; at short range ($r \ll r_*$), logarithmic:

$$V(r/r_*) \approx -\ln \left( \frac{e^\gamma r}{2r_0} \right),$$  \hspace{1cm} (16)

where $\gamma \approx 0.57721$ is Euler’s constant [98].

We do not include contact interactions between charge carriers due to exchange and correlation effects that occur when they are localized on the same site [89], since these partially cancel out of BEs for complexes larger than X. We consider e to have the same mass in the spin-up and spin-down conduction bands of 1L semiconductors. To test this, we calculate the sensitivity of quinton BEs to the electron mass for 1L-WS₂. We use the h mass as for Table I, but change the spin-down and spin-up e masses to be 10% lighter and 10% heavier than the e mass of 1L-WS₂ in Table I. This is a fair assumption, as Ref. [7] calculated that e in the upper spin-split CBs in 1L-TMDs are ~ 20% lighter than lower spin-split CBs. Our results show that the quinton binding energy of 1L-WS₂ with different e mass is ~ 59 meV, similar to the ~ 57.4 meV in Table I, also consistent with Table VI of Ref. [29].

B. QMC calculations

We use VMC [85] and DMC [32,99] to calculate the total energies of complexes of charge carriers in 1L-LSMs. We use the RKI potential in Eq. (15) or, for the short range ($r \ll r_*$) limit, the logarithmic interaction of Eq. (16). Our trial wave functions for complexes of distinguishable charge carriers are of the Jastrow form [21], which includes a pairwise sum of terms depending on the distances between charge carriers, as for Ref. [29]. Trial wave functions are optimized within VMC by minimizing first the energy variance [100,101], then the energy expectation [85]. Our fixed-node DMC energies are exact solutions to the Mott-Wannier-Keldysh model of Eq. (14). DMC calculations use time steps in the ratio 1 : 4, with the corresponding target configuration populations in the ratio 4 : 1. The resulting energies are extrapolated linearly to
zero time step and to infinite population. QMC calculations are done with the CASINO code [21].

C. Fitting function for BE as a function of magnetic field

We consider a complex of \( N_c \) and \( N_h \) e and h interacting via the logarithmic approximation to the Keldysh interaction in the presence of a uniform magnetic field \( \mathbf{B} = (0, 0, B) \). Let \( \tilde{B} = B/B_0, \tilde{m}_i = m_i/\mu, \tilde{q}_i = q_i/e, \tilde{r}_i = r_i/(\sqrt{2}r_0) \), and \( \tilde{r}_s = r_s/(\sqrt{2}r_0) \) be magnetic field, mass, charge, and position of particle \( i \). The screening length and the Hamiltonian \( \tilde{H} = \tilde{H}/E_0 \) in logarithmic e.u. are as defined in Sec.II A. We thus get:

\[
\tilde{H} = \sum_i \frac{1}{2\tilde{m}_i} \tilde{r}_i^2 + \sum_i \frac{\tilde{B}^2 \tilde{r}_i^2}{8\tilde{m}_i} - \sum_{i>j} \tilde{q}_i \tilde{q}_j \ln (e^\gamma \tilde{r}_{ij}/2) + \sum_{i>j} \tilde{q}_i \tilde{q}_j \ln (\tilde{r}_s),
\]

where we neglect the term \((\hbar \tilde{q}_i/\tilde{m}_i) \mathbf{A}_i \cdot \nabla_i \) in Eq. (10) that breaks time-reversal symmetry. The energy eigenvalue \( \tilde{E} = E/E_0 \) is therefore the sum of a function \( f(\sigma, \tilde{B}) \), where \( \sigma = m_c/m_h \), and an additive constant \( c(\tilde{r}_s) = \sum_{i>j} \tilde{q}_i \tilde{q}_j \ln (\tilde{r}_s) \).

For \( \tilde{B} \) such that the magnetic confinement energy is larger than the log interaction, the interaction \(-\sum_{i>j} \tilde{q}_i \tilde{q}_j \ln (e^\gamma \tilde{r}_{ij}/2) \) is negligible compared with the magnetic confinement energy of each particle. The dimensionless total energy is the sum of the zero-point energies of the individual particles in the quadratic potential plus the constant \( c(\tilde{r}_s) \). Hence, at large \( \tilde{B} \gg 1 \):

\[
\tilde{E} = \left( \frac{N_c}{\tilde{m}_c} + \frac{N_h}{\tilde{m}_h} \right) \frac{\tilde{B}}{2} + O(1) + c(\tilde{r}_s)
\]

\[
\approx \left( \frac{N_c}{\tilde{m}_c} + \frac{N_h}{\tilde{m}_h} \right) \frac{\tilde{B}}{2} + \tilde{E}_{\tilde{B}=0},
\]

since \( \tilde{E}_{\tilde{B}=0} \sim c(\tilde{r}_s) \), when \( \tilde{r}_s \) is large (\( \tilde{r}_s \gg 1 \)).

For small \( \tilde{B} \ll 1 \), we use the CoM zero-point energy approximation, Eq. (12), in which we assume the quadratic potential varies on the scale of the complex. Then:

\[
E = \frac{\tilde{B}}{2} \sqrt{\frac{N_c/\tilde{m}_c + N_h/\tilde{m}_h}{N_c\tilde{m}_c + N_h\tilde{m}_h + \tilde{E}_{\tilde{B}=0}}}
\]

The total energies for X with \( m_c = m_h \) are calculated using the finite-element method implemented in MATHEMATICA [102]. The results are converged by increasing the region size and decreasing the maximum cell size in order to achieve at least six digits of precision. This leads to errors comparable errors to QMC (see Sec.IV B). Subtracting the large-\( \tilde{B} \) Eq. (19), from the energy shift of X due to external magnetic fields, results in the logarithmic-like behavior in Fig. 9. This suggests the following formula for the energy shift of a generic charge-carrier complex due to external magnetic field:

\[
E - \tilde{E}_{\tilde{B}=0} = \frac{1}{2} \left[ \frac{N_c/\tilde{m}_c + N_h/\tilde{m}_h}{N_c\tilde{m}_c + N_h\tilde{m}_h + \tilde{E}_{\tilde{B}=0}} \right] \ln(1 + \tilde{B} + C^2 \tilde{B}^2) + \left( \frac{N_c}{\tilde{m}_c} + \frac{N_h}{\tilde{m}_h} \right) \frac{\tilde{B}^2}{2}.
\]

where \( C = C(\tilde{r}_s, \sigma) \) is independent of \( \tilde{r}_s \) in the \( r \ll r_s \) limit in which the logarithmic interaction is valid.

We use the least-squares method to fit the finite-element method results for the BE shift of X with equal \( m_c^* \) and \( m_h^* \), Fig. 9, for several values of susceptibility, and extract the

![FIG. 9. (a) Dependence of \( C_X \) on susceptibility. The markers show fitted \( C_X \). The line is a quadratic fit to the points as for Eq. (22). (b) Shift in energy of X with equal \( m_c^* \) and \( m_h^* \) due to external magnetic field, after subtracting the large-\( \tilde{B} \) behavior from Eq. (19), for several \( \tilde{r}_s \). Markers indicate finite-element method results, while lines show the fit of Eq. (21).](image-url)
fitting parameter $C_X$ for each $\tilde{r}_x$. We use a polynomial fit to get the dependence of $C_X$ on susceptibility, Eq. 9:

$$C_X = -0.1020(22) + 0.5469(9)x + 0.1946(4)x^2,$$

where $x = \tilde{r}_x/(1 + \tilde{r}_x)$. Since most 1L-TMDs have effective mass ratios close to 1, Table I, we neglect the mass-ratio dependence of $C$. Our fit is only valid for $\tilde{r}_x \gtrsim 0.5$. Although the fit from Eq. (21) is derived for the logarithmic interaction, it does fit well our DMC results in Fig. 6 for the full Keldysh interaction for experimentally relevant values, as shown by the red curves in Fig. 6.

**ACKNOWLEDGMENTS**

We thank M. Aghajanian, A. A. Mostofi, J. Lischner, V. I. Fal’ko, G. Wang for useful discussions. We acknowledge support from EPSRC Grants No. EP/P010180/1, No. EP/L01548X/1, No. EP/K01711X/1, No. EP/K017144/1, No. EP/N010345/1, No. EP/L016087/1, No. EP/X015742/1, No. EP/V000055/1, ERC grants Corr-NEQM (Grant Agreement No. 853368), Hetero2D, GSYCOR, GIPT, CHARM, Graph-X, Lancaster University’s High-End Computing facility, EU Graphene and Quantum Flagship and Graph-X.

[69] Download Executable_program.zip file from https://doi.org/10.17863/CAM.87211 and follow the instruction provided in README.txt to run the software.


